pair moved as a unit. The loading blocks were mounted in the center space by a 0.050-inch brass supporting tube clamped in the top of the cavity. Four blocks were tested (Fig. 2): one with a square cross section to give nearly equal frequencies, one with rectangular cross section to give moderate separation of frequencies, one *I* shaped block to give extreme separations, and the block shown in Fig. 2(d). The large horizontal dimensions of this last block make the space between block and cavity walls small, thereby providing high capacitative loading. The holes in the corners help conserve the effective inductance by allowing the magnetic field a near normal path. Each block had a vertical dimension of 0.280 inch (*i.e.*, about one third of the depth of the cavity). The other dimensions are shown in Fig. 2.

Power was introduced by two magnetic coupling loops which could be rotated or retracted. The two loops could be used to feed each mode independently, or one loop could be oriented to feed both and the other loop left as a tuning monitor.

Measured Q values corrected for coupling losses were 2000 to 3000. Tuning curves for the cavity when loaded with the rectangular block are shown in Fig. 3. Results for this and other blocks are summarized in Table I.

Ceramic plungers with a thickness of 0.120 inch (*i.e.*, two times the original thickness) were fitted and tested with the square cross section block [Fig. 2(a)]. This resulted in a tuning range from 5655 Mc to 7450 Mc for each mode. A block of quartz (relative dielectric constant 4) with a volume of 0.034 cubic inch was placed in the bottom of the cavity to estimate the amount of detuning likely to be caused by the presence of a paramagnetic or other sample in the cavity. The quartz occupied about one fourth of the total volume available for the sample and lowered the cavity frequency by less than one per cent.

Capacitative loading by centrally mounted copper blocks is free from lossy joints between conducting elements and does not lead to an excessive deterioration in Q. In magnetic resonance experiments, there may be an over-all gain in sensitivity due to the reduction in cavity volume and the increase in sample filling factor. For a given size cavity the operating frequencies may be chosen anywhere in a wide range and may be altered merely by substituting new loading blocks. Because of the effect of the block in concentrating the electric field between itself and the cavity wall, it is possible to obtain a fair degree of independence in the tuning of the two modes and to cover a range up to twenty per cent without using large amounts of dielectric. The lower third of the cavity volume is left free for the mounting of samples and contains the region of strong magnetic field common to both modes.

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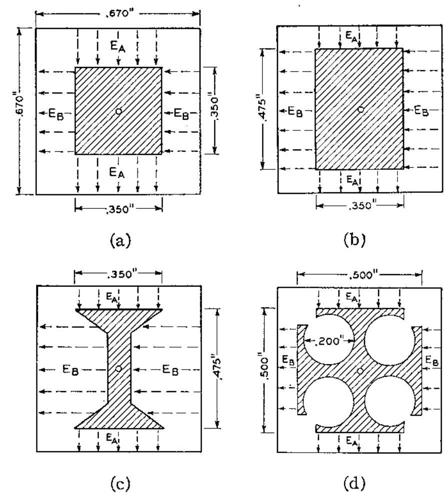
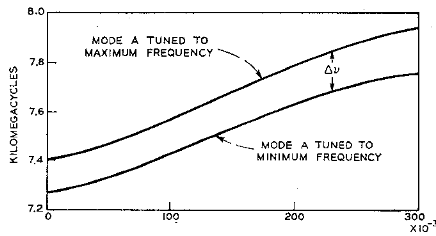
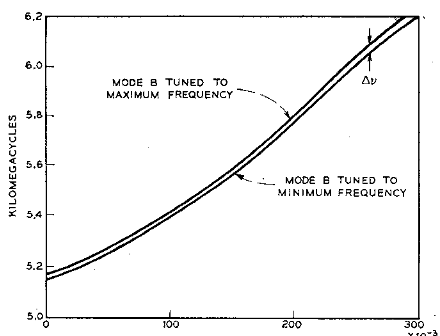


Fig. 2—Plan views of cavity with each loading block in place. E_A and E_B indicate the direction of electric field for modes A and B. Third dimension of all blocks in 0.280 inch.



(a) Displacement of mode B tuning plungers in inches.



(b) Displacement of mode A tuning plungers in inches.

Fig. 3—Tuning curves for each cavity mode when loaded with rectangular block [Fig. 2(b)]. Tuning plunger displacements are measured from the central position (*i.e.*, the position in which the plunger occupies the space between the loading block and the side wall). The two curves shown in each case correspond to the two extreme tuning settings in the remaining mode. Δv is a measure of the independence of tuning.

TABLE I*

Type	High End	Δv	Low End	Δv	Tuning Range
Square Block Modes A and B	7700 Mc	50 Mc	7075 Mc	40 Mc	625 Mc
Rectangular Block Mode A Mode B	6220 Mc 7940 Mc	25 Mc 190 Mc	5150 Mc 7265 Mc	20 Mc 135 Mc	1070 Mc 675 Mc
I Block Mode A Mode B	5280 Mc 9150 Mc	25 Mc 305 Mc	4400 Mc 8500 Mc	2 Mc 255 Mc	880 Mc 650 Mc
Block of Fig. 2(d) Modes A and B	4950 Mc	30 Mc	3850 Mc	50 Mc	1100 Mc

* Δv is a measure of independence of tuning. It is the change in frequency of one mode when the tuning control for the other mode is taken from one extreme end to the other. See Fig. 3.

Unloaded Q of Single Crystal Yttrium-Iron-Garnet Resonator as a Function of Frequency*

The practical feasibility of constructing magnetically tunable broad-tuning range microwave filters using single crystal yttrium-iron-garnet resonators was demonstrated in a recent paper.[1] Experimental results were presented on one- and two-resonator filters which can be tuned by varying a dc magnetic field bias over a full waveguide bandwidth and greater, at the same time maintaining an insertion loss performance which is comparable to mechanically-tuned cavity filters. The crucial parameter of the resonant elements in a band-pass filter is the unloaded Q, Q_u. With a spherical single crystal of yttrium-iron-garnet the Q_u decreases with frequency below X-band frequencies reaching very low values at frequencies around 2000 Mc.

Analytical formulas for Q_u ($=2\pi\times$frequency$\times$total energy stored/power absorbed at resonance) have been developed.[2] First, formulas given by Lax[3] were used for the effective susceptibility, which relates the RF components of magnetization inside a ferrite to the *external* RF fields. Lax uses the original Landau-Lifshitz formulation of the equations of motion and makes the substitution $\alpha=1/\omega\tau$ for the original damping parameter α, where $\tau=a$ relaxation time, and $\omega=2\pi\times$frequency. By using his susceptibility formula the following expression for Q_u of a sphere was obtained:

$$Q_u = \omega_0\tau/2 \text{ (Lax)}. \tag{1}$$

Recently an analytical formula for the effective susceptibility was developed[4] using the modified form of the Bloch equations of motion of magnetization which were given by Bloembergen.[5] Using this result a new re-

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[1] P. S. Carter, Jr., "Magnetically Tunable Microwave Filters Employing Single Crystal Garnet Resonators," IRE 1960 International Convention Record, pt. 3, pp. 130–135.

[2] P. S. Carter, Jr., *et al.*, "Design Criteria for Microwave Filters and Coupling Structures," Stanford Res. Inst., Menlo Park, Calif., Tech., Rept. No. 8, SRI Project 2326, Contract DA 36-039 SC-74862; October, 1959.

[3] B. Lax, "Frequency and loss characteristics of microwave ferrite devices," Proc. IRE, vol. 44, pp. 1368–1386; October, 1956.

[4] C. Flammer, "Resonance Phenomena in Ferrites," unpublished memorandum.

[5] N. Bloenbergen, "Magnetic resonance in ferrites," Proc. IRE, vol. 44, pp. 1259–1269; October, 1956. Eq. (5) (Bloch-Bloenbergen equations of motion) contains an error. The term $-M_0/\tau$ should be deleted.

lation was then obtained for Q_u:

$$Q_u = \left(\omega_0 - \frac{\omega_m}{3}\right)\frac{\tau}{2} \qquad (2)$$

where

τ = relaxation time

ω_0 = resonant angular frequency $= 2\pi \times$ resonant frequency in cycles per second

$\omega_m = \gamma_0 M_0 = \mu_0 g(e/2m)M_0 = 2\pi f_m$ = angular frequency corresponding to the magnetization, in which

μ_0 = intrinsic permeability of free space $= 1.256\times10^{-6}$ henries per meter

g = Landé g factor $\cong 2.00$ for electrons in most ferrites

e/m = ratio of charge, e, to mass, m, of electron $= 1.759\times10^{11}$ coulombs/kg

M_0 = saturation magnetization of ferrite, amperes per meter.

This new formula for Q_u given above predicts that a value of $Q_u=0$ should occur at $\omega_0=\omega_m/3$, thereafter increasing linearly with increasing frequency, provided that τ is independent of frequency. According to the equation for Q_u derived using the Lax formula, the variation of Q_u is described by a straight line which intersects the origin, $Q_u=0$ at $f_0=0$. These two relations given by (1) and (2) are shown in Fig. 1 for the case $\tau=2\times10^{-7}$. The only qualification that must be applied to these formulas is that the material must be fully magnetized. For a spherical shape this requires that the operating frequency should be somewhat greater than $\omega_0=\omega_m/3=\gamma_0 H_0$, since a sphere becomes unsaturated at biasing fields of this magnitude For single crystal yttrium-iron-garnet this "saturation frequency" occurs at $f_0=f_m/3=1670$ Mc.

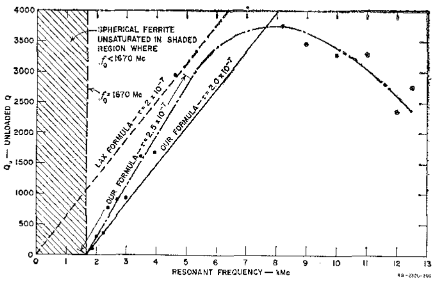

Fig. 1.

It is possible to reinterpret Lax's susceptibility formula so that it now becomes the same as the new formula. This is done by substituting for τ in Lax's equation a new relaxation time $(\omega_0 - N_z\omega_m)\tau/\omega_0$, where N_z is the z-demagnetizing factor. In the case of the sphere, $N_z=\frac{1}{3}$. However, the new formula is obtained straightforwardly from the Bloch-Bloembergen equation of motion, and the equivalent circuit interpretation, without the artificial introduction of a relaxation time which in turn depends on a demagnetizing factor.

Measurements were made of the Q_u of a highly polished spherical single crystal of yttrium-iron-garnet, using the method described by Ginzton.[6] The yttrium-iron-garnet sphere was mounted in a short-circuited waveguide or transmission line near the short circuited end. A 0.064-inch diameter single crystal yttrium-iron-garnet sphere was used in these measurements.

The experimental values of Q_u are shown as points in Fig. 1. An approximate fit to these data is given by the "dash-dot" curve. The lower frequency portion of this experimental curve between 1.67 kMc and about 5 kMc is a straight line which can be represented by the new formula, assuming $\tau=2.5\times10^{-7}$. In this low frequency region at least, the data appear to support the new formula for Q_u. At higher frequencies the experimental curve for Q_u flattens off and, above about 8 kMc, Q_u decreases with increasing frequency.

It is planned to publish a complete analysis and discussion of these and other related data in the near future.

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[6] Edward L. Ginzton, "Microwave Measurements," in "Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y., ch. 9, pp. 391–434; 1957.

A Note on the Derivation of the Fields in a Radial Line*

The concept of a radial transmission line is frequently used in the description of such devices as cylindrical cavity resonators and horn radiators. An approach[1–3] to the problem of determining the electric and magnetic fields in the radial line has been to solve Maxwell's equations in component form with appropriate boundary conditions. While the following derivation yields nothing new, it does, however, have the advantages of being simple and of requiring a minimum of guess work as compared to other methods of solving this problem.

The technique employed here is based on the fact[4] that the general solution of the vector Helmholtz equation

$$\nabla^2\bar{A}(\bar{r}) + k^2\bar{A}(\bar{r}) = 0 \qquad (1)$$

consists of a linear combination of three vector functions generated in turn from three scalar functions:

$$\bar{L}(\bar{r}) = \nabla\phi(\bar{r})$$
$$\bar{M}(\bar{r}) = \nabla\times[\bar{u}\psi(\bar{r})]$$
$$\bar{N}(\bar{r}) = \frac{1}{k}\nabla\times\nabla\times[\bar{u}\chi(\bar{r})].$$

The vector $\bar{u}$ is a constant vector and ϕ, ψ and χ are each solutions of the scalar Helmholtz equation; *e.g.*, $\nabla^2\phi(\bar{r})+k^2\phi(\bar{r})=0$.

Consider the geometry shown in Fig. 1. The region of interest is the semi-infinite space between the perfectly conducting, parallel bounding surfaces at $z=0$ and $z=b$. Assume that the fields have a time dependence of the form $e^{j\omega t}$ and that no free charge exists in the region between the bounding surfaces. Subject to these conditions, the electric and magnetic fields in the region $0\le z\le b$ must satisfy an equation of the same form as (1) with $k^2=-\gamma_0^2=\omega^2\mu\epsilon[1-j(\sigma/\omega\epsilon)]$ where σ, ϵ and μ are respectively the conductivity, permittivity and permeability of the medium between the surfaces and ω is the radian frequency.

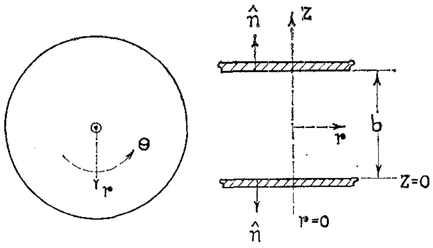

Fig. 1—A radial transmission line consisting of two parallel conducting planes.

Because of the manner in which they are defined, $\bar{M}$ and $\bar{N}$ are solenoidal as are $\bar{E}$ and $\bar{H}$ in this case, and it follows that the $\bar{M}$ and $\bar{N}$ solutions for (1) could correspond to either the electric or the magnetic field depending upon the choice of boundary conditions at $z=0$ and $z=b$.

As an illustration let us require that $\bar{M}$ and $\bar{N}$ satisfy the boundary conditions for the electric field, namely, $\hat{n}\times\bar{M}=\hat{n}\times\bar{N}=0$. Writing the $\bar{M}$ solution in terms of the unit vector in the z-direction, $\bar{M}=\nabla\times[\bar{u}_z\psi(r,\theta,z)]$, and applying the boundary conditions after solving the scalar Helmholtz equation by the standard approach of separating variables leads to

$$\bar{E}_{m,n} = \bar{M}_{m,n} = \nabla\times\left\{\bar{u}_z[K_1J_m(\beta r) + K_2Y_m(\beta r)]e^{\pm jm\theta}\sin\left(\frac{n\pi}{b}z\right)\right\} \qquad m, n = 0, 1, 2\cdots. \qquad (2)$$

K_1 and K_2 are arbitrary constants which specify the amplitude of the field. J_m and Y_m are the Bessel functions of the first and second kind respectively and $\beta^2=-\gamma_0^2-(n\pi/b)^2$. In this case, K_2 must be equal to zero because of the singularity of $Y_m(\beta r)$ at $r=0$, but if the region of interest is that for which $r\ge r_0\ne0$, then K_2 need not be zero. The corresponding magnetic field can be found from

$$\nabla\times\bar{E}_{m,n} = -j\omega\mu\bar{H}_{m,n}. \qquad (3)$$

* Received by the PGMTT, June 10, 1960. This work was supported in part by the U. S. Army Signal Engrg. Labs., Fort Monmouth, N. J., under Contract DA 36-039 sc 78254.

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